

# Wave-Length Standards in the First Spectrum of Krypton

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# I—Wave-length Standards in the First Spectrum of Krypton

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[Plate 1]

#### Introduction

A preliminary investigation of the first spectrum of Krypton was made by the author in 1931 and 1932,\* to investigate the suitability of its lines for providing wave-length standards of the highest accuracy. The ten strongest lines in the violet were measured interferometrically, and these preliminary results indicated that the spectrum of krypton was eminently suited to give a system of standard wave-The lines are extremely sharp, easily reproducible, and their distribution is such that it is possible to evaluate the thickness of an étalon without ambiguity, even when the roughly estimated thickness may be in error by several hundred The spectrum has the advantage that the brightest lines are in the violet part of the spectrum, which eliminates the necessity of using panchromatic plates for photographing them, and they are of such great intensity that their photography takes only about one-tenth of the time required for the primary standard.

It has been my practice in the last four years to photograph the spectrum of krypton before and after any spectrum compared directly with cadmium as a check on the constancy of the étalon and for the purpose of determining the whole number of waves contained by the interferometer. The experience thus gained convinced me that the spectrum is especially suitable for use as a system of secondary standards, and it was accordingly thought desirable to make a thorough interferometric investigation of the hyperfine structure, pressure shift, and reproducibility of the wave-lengths of its lines. The wave-lengths which I found in the preliminary investigation made in 1932, were used in conjunction with those found by Meggers† and Humphreys‡ for defining the I.A. standards of krypton recommended in 1932. These have the interesting distinction of being the first secondary standards which have been assigned 8-figure wave-lengths by the I.A.U.

However, it was considered necessary to redetermine the wave-lengths of the krypton lines, partly because the range of thickness of étalons previously used (1 cm

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* C. V. Jackson, 'Proc. Roy. Soc.,' A, vol. 138, p. 147 (1932).
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<sup>&#</sup>x27;Bur. Stand. Sci. Pap.,' No. 414 (1921).

<sup>&#</sup>x27;Bur. Stand. J. Res.,' vol. 5, p. 1041 (1930).

to 3 cm) was inadequate and partly because some of the determinations were made with a type of étalon (the Hilger-Williams variable gap étalon) which could not be made to give results of the same order of accuracy as that obtainable with fixed étalons.

In the new investigation I have measured the wave-lengths of the krypton lines with étalon separations ranging from  $\frac{1}{2}$  cm to 10 cm and extended the wave-length range from 3424A to 6456A. The extension of the wave-length range was made because the fainter lines at the red and ultra-violet ends of the spectrum are extremely useful for determining the phase correction of interferometer plates, although they are, perhaps, too faint for use as actual secondary standards. The extension of the range of étalon separations was made largely on account of the fluctuations of apparent wave-lengths which I had found for neon,\* when the lines of this element were examined with interferometers of varying thickness and resolving power. With the krypton lines no measureable change in apparent wave-length could be observed over the whole range of  $\frac{1}{2}$  cm to 10 cm étalon thickness. This may be taken as very strong evidence that the efficiency of the krypton lines is in no way impaired by the presence of any slight hyperfine structure, and further, that the red line of cadmium itself has a wave-length which is completely independent of instrumental It is necessary to control the conditions of producing this line very carefully, if the utmost degree of accuracy is desired.

In order to obtain the highest degree of accuracy possible in comparing the wave-lengths of krypton with the primary standard it was found necessary to design a new type of étalon carrier and two new types of étalons.

It is perhaps of interest to mention that the final wave-lengths given in this communication have been derived from the measurement of about 120 plates, all of which were direct comparisons with the primary standard. In addition, a large number of plates was taken for the purpose of investigating the hyperfine structure, pressure shifts, and the relation between the wave-lengths of krypton in tubes viewed end-on or transversely.

Incidentally, it was found necessary to investigate the correct conditions for the accurate reproduction of the primary standard itself. Briefly it was found to be essential to observe several restrictions in the use of the Michelson lamp. of the capillary must not be less than 2 mm in diameter, the temperature of the furnace must not exceed 320° C., and the current must not be greater than It is an advantage to use a tube having a volume not much less than The sharpness of the fringes at large retardations is much improved by the addition of 1 mm Hg. of air to the Michelson tube, without making any measurable difference in its wave-length. Experience with the new Osram G.E.C. cadmium vapour lamp showed that this source, when run with a current not exceeding  $1 \cdot 1$  amp gives the red line as sharply as the air-filled Michelson tube. sources were found to have wave-lengths reproducible and identical with each other within the experimental error, which was of the order of  $\pm 0.0001$ A.

<sup>\*</sup> C. V. Jackson, 'Proc. Roy. Soc.,' A, vol. 143, 1933.

The work has occupied most of my time for the last three or four years, which seems, perhaps, an excessive amount of time to expend on the determination of 50 wave-length standards, but the extreme suitability of the krypton lines for giving a system of standard wave-lengths, seemed to justify a really thorough investigation of their properties.

#### EXPERIMENTAL

About 120 direct comparisons with the primary standard were made using étalons of thickness  $\frac{1}{2}$ , 1, 2, 3, 5, 6,  $7\frac{1}{2}$ , and 10 cm. The interferometer plates were coated with sputtered silver, evaporated silver, sputtered platinum, or evaporated aluminium. Since the variable gap étalon had been found so unsatisfactory all the comparisons were made with the aid of fixed étalons of quartz.

The shorter separators (up to 3 cm) were of the type used in my previous work on neon, i.e., a tube of silica with three projecting studs at each end opposite each other. These studs have plain parallel faces and are nearly equidistant, but it is necessary to adjust the plates to exact parallelism by applying pressure with adjustable springs. This type of separator is very satisfactory with the short and medium gaps, and is extremely stable. Etalons of this type remain in perfect adjustment for many weeks, but with long gaps they are somewhat unsatisfactory on account of the difficulty of adjusting them with the necessary accuracy.

For this reason two new types of fixed étalon were designed. The first type which was designed in conjunction with my brother,\* is made entirely of fused silica, and consists of two bars which have parallel ends and are equal in length to within 1/20th The interferometer plates are fixed to the ends of these bars by of a wave-length. placing them in optical contact. These étalons were constructed by Adam Hilger, Ltd., and when they were tested it was found that they came well up to the standard of accuracy which had been specified. The only objection to this type of étalon is that it is necessary to have a separate pair of interferometer plates for each étalon, as once the quartz has been put into optical contact it is impossible to separate the pieces without grave risk of breaking them. However, it is hardly necessary to emphasize how advantageous it is to have an étalon which is permanently in exact adjustment.

The other type of separator consists of a tube of silica whose walls are a little over The ends of this tube are worked plain and parallel over their whole surfaces, again to within one twentieth of a wave-length. In order to use this type of separator it is only necessary to wring on to each end an interferometer plate, coated with any desired metallic film. Water, alcohol, or paraffin was used for wringing on the plates and they are quite satisfactory. These separators were also made by Adam Hilger, Ltd., and when tested, after wringing on the interferometer plates, it was found that their parallelism was as good as that obtainable with the optically contacted étalons. This shows that there is no loss in accuracy of parallelism

<sup>\*</sup> See Jackson and Kuhn, 'Proc. Roy. Soc.,' A, vol. 148, p. 335 (1935).

in the process of wringing on the plates, instead of placing them in optical contact. The process of wringing on the plates needs a little practice, but provided the plates and separator are perfectly clean it is quite easy. For cleaning, a mixture of pure alcohol (3 parts), ether (1 part), and ammonia (1 part, 0.880) is very effective. Apart from its rather high cost this type of étalon is almost certainly the most satisfactory.

A further improvement in the apparatus used in this investigation over that used in my previous work was in the étalon carrier. I had found that the original carrier\* left much to be desired, particularly on account of the great difficulty of obtaining accurate alignment, and for this reason a new étalon carrier was designed specially for the purpose of making wave-length comparisons. This apparatus worked so well, and is so convenient in use, that it is desirable to give a description of its principles.

When a wave-length comparison is being made by means of an étalon, it is very important that the part of the interferometer plates used should be completely and uniformly illuminated by each of the sources being compared. If this were not so inaccurate wave-lengths would result, because in practice it is impossible to adjust an étalon to less than a fortieth of a wave, while it is undesirable to have differences greater than 1 or 2 thousandths of a fringe between the optical paths transversed by the light of the sources being compared. Clearly this condition will only be satisfied if the étalon is illuminated in the same way by each source.

If the two sources which are being compared are of uniform intensity over a fairly large area (e.g., two Geissler tubes of fairly large bore) the necessary uniformity of illumination may be obtained simply by projecting enlarged images on to a diaphragm placed in front of the étalon. This diaphragm should be immediately in front of the étalon, and at such a distance from the objective of the rings that a real image of it is focused upon the prism (or grating) of the auxiliary spectrograph. With other sources, however (e.g., those which are of uneven intensity or so small that the enlarged image does not cover a reasonable area of the étalon plates), it is necessary to use a collimator in front of the étalon. The mounting, therefore, should permit the use of both of these methods of illumination.

In practice it is necessary to use lenses of different foci for projecting the rings, according to the length of the étalon used, and for collimating, and when once the apparatus has been set up it should be possible to change quickly from one lens to another without making any further adjustment for alignment. It is also desirable that when the focal positions of the lenses, slit, and étalon carrier have been determined once, they may be replaced accurately in these positions by scale, without making new adjustments.

The étalon carrier itself should have three adjustments controlled by screw The étalon must be rotatable over small angles in a vertical and horizontal

<sup>\*</sup> This was an early form of étalon (and Lummer plate) carrier, designed, and manufactured by Adam Hilger, Ltd.

plane for the purpose of centring the ring system on the slit of the spectograph, and also it should be capable of being raised or lowered (without interfering with any of the other adjustments) to bring the centre of the étalon to the same height as the optical axis. This last adjustment is necessary because it is not always convenient to use étalons of exactly the same dimensions.

THE FIRST SPECTRUM OF KRYPTON

It is thought that the instrument shown in fig. 1 fulfils all these conditions without introducing any unnecessary complications. It was constructed to my design by Adam Hilger, Ltd. The base of the instrument consists of a girder G of triangular This is supported at the end nearer the spectrograph by a cup and The cup which carries the ball is provided with the necessary adjustments

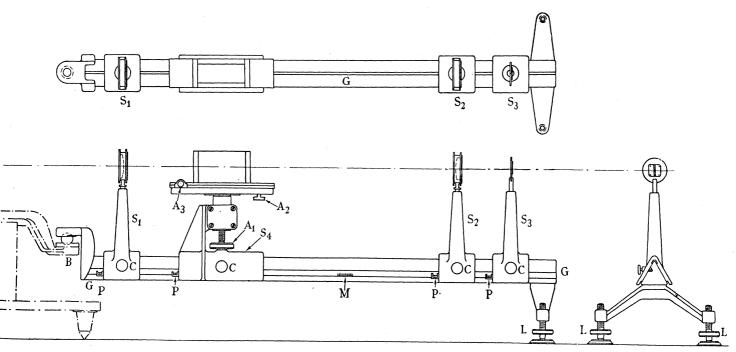


Fig. 1.

for setting its position both in a vertical and in a horizontal plane, so that this end of the apparatus may be easily set in correct alignment with the optical axis of the spectrograph. The other end is supported by the levelling screws L by means of which it may be raised to the correct height. The lateral adjustment of this end is made by moving it with a suitable screw, across the surface of the table which carries the whole apparatus. The objective of the rings and the collimator objective are supported by the stands S1 and S2 respectively. The bases of these stands are in the form of saddles, which fit accurately on the triangular girder, so that they may be moved along the whole length of the girder without upsetting the alignment of the axis of the lens they are carrying. The slit is on a similar carrier S3. Each is provided with a clamping screw at C, and a pointer P, which shows their position on the millimeter scale M, which is engraved along the whole length of the scale.

The method of support of the étalon carrier can be seen in fig. 1. Rotation of the screw A1 raises or lowers the carrier, without altering the angle made by the étalon with the optical axis; this adjustment is very useful for testing the parallelism of The angular adjustment of the étalon is made in a horizontal and vertical sense, respectively by the screws A2 and A3. The instrument is shown with the 10 cm contacted étalon in position.

The objectives of the rings are quartz fluorite, which forms a very achromatic combination, and have foci 6-in, 13-in, and 20-in. The 13-in lens was used for those taken with étalon separations of 5 mm and less, while the 20-in lens was used for some of the photographs taken with the three longest étalons. It was found, however, that there was no gain in accuracy by using the 20-in lens even with the longest étalon.

The mounting of the objectives in their stands is such that they may be quickly interchanged, without any loss of accuracy of their centring and without undoing any screws. It was designed by Adam Hilger, Ltd., for use on one of their spectro-The diaphragm is placed on the étalon carrier immediately in front of the front plate of the étalon; it is of the iris type and adjustable in diameter from 1 mm to 20 mm. Most of the photographs of the krypton spectrum were taken without the collimator, as both the sources of krypton and cadmium are suitable for this method of comparison. It has the advantage over the collimator method that the exposure is considerably shorter.

For nearly all the comparisons Ilford hypersensitive plates were used, on account of their fine grain and extreme rapidity to Cd 6438. For some of the plates (e.g., the hyperfine structure and pressure shift plates) Ilford "Monarch" plates were used as these are more rapid to the violet than the hypersensitive plates.

The light source was placed about 8 feet away from the interferometer and the light was condensed on the étalon (or the collimator slit) by a quartz objective of 6-in aperture and 15-in focus.

All the photographs were taken by the method of alternate exposure, e.g., Cd 2 min, Kr 2 min, Cd 2 min, . . ., and before and after each comparison a short exposure of krypton was taken on a different part of the plate, for detecting any change that might occur in the optical length of the étalon during the comparison. The change was rarely found (even with the longest étalon) to exceed a few thousandths of a fringe during a comparison; in fact, it was nearly always less than the setting errors on the fringes. For example, in a series of comparisons using the 10 cm étalon, the value of e for the cadmium line was found to be 0.424, 0.416, and 0.408, although a period of about half an hour elapsed between successive exposures. The change during the exposure must have been considerably less than this, probably of the order of 0.003 of a fringe, which is slightly more than 1 in 100,000,000. constancy in the optical length of the étalons is due to the efficiency of the temperature control in the Equable Temperature Room of the Imperial College, and of the heat installation of the cadmium furnace. All the controlling rheostats were outside the Equable Temperature Room, and any effect of the radiation from the furnace was

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minimized by its distance from the interferometer. Since change in barometric pressure has a very deleterious effect on the constancy of the optical length of an étalon no comparisons were made on days when the barometer was falling or rising rapidly.

It should perhaps be mentioned that invar separators have the advantage over those made of silica, since the thermal expansion of their length almost exactly compensates the alteration in optical length caused by the change of the refractive index of air when the temperature varies.

The krypton tubes were either obtained from Professor Lepape or prepared by The Michelson tubes which were all prepared in the laboratory were used either with or without the addition of 1 mm Hg or air, except the 7½ cm and 10 cm étalons, for which air-filled tubes were always used. A few of the comparisons were made against the Osram lamp (current =  $1 \cdot 1$  amp). Both the cadmium and krypton tubes were excited by means of a small Marconi transformer, the current ranging from 5 to 20 milli-amperes.

The phase correction of the various interferometer plates used was determined by measuring the apparent wave-lengths of the krypton lines with a very short separator of about 1.3 mm gap. The separator in this case consisted of 3 small ball bearings, which were sufficiently accurate to require only quite small pressure on the adjusting screws to make the étalon plates parallel. The difference between the apparent wave-lengths thus determined and the true wave-lengths gives the phase correction for a gap of 1.3 mm, and the phase correction for an étalon of gap N mm is naturally 1.3 x/N, where x is that for the 1.3 mm étalon. If the error of the wave-length as determined by this short étalon is 0.002A (about the usual error) that for the 5 cm étalon will be only 0.00005A. It is evident that standard wave-lengths required only for the purpose of determining phase corrections are entirely adequate if they are known only with an accuracy of  $\pm 0.001$ A. For this reason the few faint krypton lines in the ultra violet and red, whose wave-lengths may have errors of nearly that amount are very useful standards for this purpose. The phase corrections determined with the short étalon were always plotted on a graph, as this reduced the effect of accidental errors.

The method of reduction was the same as that used in my previous work. Except with the shorter étalons the first, second, and eleventh rings were measured. The object of measuring the outer ring is to obtain a more accurate value for the With short étalons it is not possible to use rings so far out as for the longer ones, because for large angles the square law is not obeyed. down to 1 cm thickness the eleventh ring may be used without introducing any appreciable error, while for  $\frac{1}{2}$  cm and  $\frac{1}{4}$  cm étalons the highest rings which should be measured are respectively the fifth and the third.

The ring scale (i.e., the difference between the squares of the diameters of two adjacent rings) should theoretically be exactly proportional to the wave-length of the line giving the rings. This was tested and it was found that, after making the necessary correction for the varying magnification of the spectrograph for different

wave-lengths (due to the tilt of the plate), this condition was fulfilled, within the error of measurement. It was also found that the scale given by the first and second rings was within experimental error equal to that given the outermost ring measured, for all the étalons used.

All the observed wave-lengths were corrected to air at 15° C and 760 mm Hg. No correction was made for the humidity of the air. This was between 40 and 60%.

#### THE PRESSURE SHIFT OF THE KRYPTON LINES

In an investigation on the wave-lengths of the neon\* lines (1s-2p multiplet), it was found that the gas in the spectrum tube could be at as high a pressure as 15 mm Hg without affecting either the sharpness or the wave-lengths of the lines. advantage of using a fairly high pressure in spectrum tubes filled with the rare gases is that their life is very much increased. A preliminary investigation of the spectrum of krypton showed that even with a pressure of 10 mm Hg in the tube, the violet lines (1s—3p multiplet) are appreciably broadened, and shifted to the red by a measurable amount. For this reason it was decided to carry out a fuller investigation of the pressure shifts of the krypton lines before redetermining their wave-lengths. For this purpose comparisons of the wave-lengths given by tubes filled at pressures of 0·1 mm, 4 mm, 10 mm, and 20 mm Hg were made.

The results of these comparisons, in which 30 lines were investigated are given in The pressure shifts are given in cm<sup>-1</sup> for tubes filled with krypton at pressures of 4, 10, and 20 mm Hg relative to the wave-length given by a tube containing krypton of pressure of 0.1 mm Hg. The shift is always to the red with increase of pressure.

It is clear from the Table of observational data that within experimental error the shift for all the 1s terms is equal at all the pressures investigated. This is naturally assumed to be zero, and it is then possible to give the pressure shifts for the other terms involved in the observed lines.

The values of the term pressure shifts are given in Table II.

The lines involved in the 1s and 2p terms only show an appreciable pressure shift at 20 mm Hg, as both with the 4 mm tube and the 10 mm tube the pressure shift is too small to measure, i.e., less than 0.0001A. With the 1s—3p lines, however, it is considerably greater, the mean value for the 3p terms being 0.003 cm<sup>-1</sup> for the 10 mm tube, while with a pressure of 20 mm it is 0.011 cm<sup>-1</sup>. For the 4 mm tube it appears to be about 0.0003 cm<sup>-1</sup> which is well under 0.0001A, i.e., less than the experimental error of the wave-length determinations of the krypton lines. interesting that in all cases the pressure shift is, within experimental error, dependent on the principal quantum number of the terms only. This applies to the 1s, 2p, 3p, and 4p terms, and probably to the 5d terms too.

<sup>\*</sup> C. V. Jackson, 'Proc. Roy. Soc.,' A, vol. 143, p. 124 (1933).

# Table I—The Observed Pressure Shifts in Krypton.

THE FIRST SPECTRUM OF KRYPTON

	0·1 mm Hg-4 mm Hg	0·1 mm Hg-10 mm	Hg 0·1 mm Hg–20 mm Hg	Terms
	$cm^{-1}$	$cm^{-1}$	cm <sup>-1</sup>	
3665	+0.007	Diffuse	Very diffuse	$1S_5 - 4p_6$
3679	+0.005	,,	, ,,	$1S_5 - 4p_8$
3773	+0.008	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	"	$1S_4 - 4p_5$
3796	+0.007	, ,,	,,	$1S_4 - 4p_6$
3800	+0.005	<b>"</b>	,,	$1S_4 - 4p_7$
3812	+0.006		,,	$1S_4 - 4p_8$
3837	+0.008	,,	, ,,	$1S_5 - 5Y$
4273	+0.0006	+0.003	+0.01	$1S_5 - 3p_6$
4282	+0.0007	Manage 1979 M	-	$1S_5 - 3p_7$
4286	+0.001		<u> </u>	$1S_5 - 5p_3$
4300	+0.001			$1S_4 - 3p_4$
4318	+0.0005			$1S_5 - 3p_8$
4319	+0.0005	+0.003	+0.01	$1S_5 - 3p_9$
4351	+0.0006		•	$1S_2 - 3p_1$
4362	$\pm 0.0000$	+0.003	+0.005	$1S_5 - 3p_{10}$
4376	+0.0005	+0.002	+0.008	$1S_4 - 3p_5$
4399	$\pm 0.0000$	+0.003	+0.015	$1S_2 - 3p_2$
4425	+0.0004	acceptance.		$1S_2 - 3p_4$
4453	+0.0003	+0.004	+0.01	$1S_4 - 3p_6$
4463	$\pm 0.0000$	+0.002	+0.008	$1S_4 - 3p_7$
4502	$\pm 0.0000$	+0.002	+0.015	$1S_5 - 3p_8$
5562	+0.0000	E-TERROPPE	+0.003	$1S_4 - 2p_2$
<b>557</b> 0	$\pm 0.0000$	accordand.	+0.003	$1S_5 - 2p_3$
5649		+0.003	+0.01	$1S_5 - 3p_{10}$
5870	$\pm 0.0000$		+0.003	$1S_4 - 2p_2$
5879		diament and the second	+0.005	$1S_4 - 2p_3$
5993	$\pm 0.0000$		+0.003	$1S_4 - 2p_4$
6056		-	+0.02	$2p_{10} - 5d_5$
6421		Total Andrews	+0.01	$2p_3 - 5d_4$
6456	**********	professional and the second	+0.01	$2p_9 -5d_4$

#### TABLE II—PRESSURE SHIFT IN TERM VALUES OF KRYPTON.

	$0 \cdot 1$ –4 mm Hg	$0 \cdot 1 - 10 \text{ mm Hg}$	$0 \cdot 1 - 20 \text{ mm Hg}$
	$cm^{-1}$	$cm^{-1}$	$cm^{-1}$
$2p_2$	$\pm 0.0000$	$\pm 0.0000$	+0.003
$2p_3$	$\pm 0.0000$	$\pm 0.000$	+0.003
2p4	±0.0000	$\pm 0.0000$	+0.003
2p	±0.0000	±0·0000	+0.003
mean			

Table II—continued.

	$0 \cdot 1$ –4 mm Hg	$0 \cdot 1 - 10 \text{ mm Hg}$	$0 \cdot 1 - 20 \text{ mm Hg}$
	cm <sup>-1</sup>	cm <sup>-1</sup>	cm <sup>-1</sup>
$3p_1$	+0.0005	-	-
$3p_2$	$\pm 0.0000$	+0.003	$+0.01_{5}$
$3p_3$	+0.0005	Materialization	
$3p_4$	+0.0008	automobile.	Annual
$3p_5$	+0.0005	$+0.002_{5}$	+0.008
$3p_{6}$	$\pm 0.0000$	+0.003	+0.01
$3p_7$	+0.0004	+0.003	+0.008
$3p_8$	+0.0002	+0.003	$+0.01_{5}$
$3p_9$	+0.0005	+0.003	+0.01
$3p_{10}$	$\pm 0.0000$	+0.003	+0.01
3p	+0.0003	+0.003	+0.011
mean			
$4p_5$	+0.008		
$4p_6$	+0.007		-
$4p_7$	+0.005	and the same of th	Springer (cold)
$4p_8$	+0.006	-	
$4p_9$	+0.005		America
4p	+0.006		
mean			
5Y	+0.008		
$5d_4$	·	-	$+0.01_2$
$5d_5$		programmy.	$+0.01_{2}$
$5d_4$			+0.02
$\overline{5d}$	-	Alaentour	+0.01,
mean			

For the 4p terms the pressure shift between tubes of 0.1 and 4 mm Hg only was measured, because at higher pressures the lines become so much broadened that they are obviously unsuitable for use as standards of wave-lengths.

It can also be seen from Table I that the pressure shift for any given term is proportional to the square of the pressure. Since this paper is not concerned with theoretical considerations of the cause of pressure shifts in krypton this point will not be further discussed. However, it is hoped that it may be useful because it can be applied to the interpolation of pressure shifts.

The data given in Tables I and II are sufficient for calculation of the pressure shift for all the lines measured as wave-length standards in this investigation, and the numerical value of the pressure shifts, between tubes run at 0·1 and 4 mm Hg, are given in the last column of Table IV.

#### THE EFFECT OF IMPURITIES

The spectrum of krypton is very sensitive to small quantities of active gases, such as oxygen, nitrogen, hydrogen, or mercury vapour. Even when they are present in very small quantities the lines of their spectra appear very much stronger than those of krypton. Indeed, it is somewhat difficult to prepare krypton tubes which give spectra entirely free from lines due to these elements. It was therefore considered advisable to determine whether the presence of impurities had any effect on the wave-lengths of the krypton lines. The test was made by comparing the wavelengths given by a tube showing only krypton lines with those from tubes showing impurity lines due to air, hydrogen, and mercury. The tubes were always prepared by deliberately neglecting the precautions necessary for eliminating the respective impurities.

Direct comparisons of the wave-lengths of the ten brightest violet lines and the three green and yellow lines were made with a 5 cm étalon. There was never any measurable difference between the wave-lengths given by the tube containing pure krypton and those containing impure krypton; although the spectrum of the impurity in all the tubes was considerably stronger than that of krypton. A difference in wave-length of 0.0001A could have been detected.

# THE RELATION BETWEEN THE WAVE-LENGTHS OF THE KRYPTON LINES WHEN OBSERVED END-ON AND TRANSVERSELY

It is possible to view the krypton light either in the end-on position or the transverse position of the tube. The advantage of viewing the tube in the end-on position is that the light is very much more intense than when it is viewed transversely, which enables the time exposure to be reduced by about 20 times. The disadvantage is that there is a risk of self-reversal or self-absorption occurring in the lines. It can be observed directly that there is no self-reversal for the visible and ultra-violet lines of krypton when the tube is viewed end-on. The fact that the exposure is so greatly reduced when the tube is viewed end-on leads one to suppose that there is at any rate no serious self-absorption for these lines.

The risk of change of wave-length through self-absorption is brought about by the isotopic nature of krypton. For if there is an isotope shift present in the lines, which is too small to be resolved, self-absorption would cause a change in the position of the centre of gravity of the lines; because the self-absorption is stronger for the components due to the elements present in the greatest proportion, and with an asymmetrical pattern would shift the centre of gravity towards that of the weaker lines.

For this reason, direct comparisons were made of the wave-lengths of all the stronger lines of krypton, when observed end-on and transversely. It was impossible to detect any measurable difference in the wave-length, though it would have been

possible to detect a difference of  $\pm 0.0001$ A. This applies even to the 1s-2p lines (except those in the infra red) which are the lines most likely to be affected by self-absorption.

It may therefore be concluded that, with the exception of certain of the infra-red lines, it is perfectly safe to use the krypton lines as wave-length standards whether the tube is viewed transversely or end-on.

#### THE HYPERFINE STRUCTURE OF THE KRYPTON LINES

Since krypton has a relatively large mass it is to be expected that its spectral lines will be very sharp. This is actually found to be so, for example, for the two green lines it is possible to get measurable fringes with path differences up to The violet lines are also very sharp although it is not possible to 600,000 waves. observe them with quite such high orders of interference.

This is very advantageous for lines which are proposed as wave-length standards, but unfortunately krypton has the disadvantage that it is a mixture of six different isotopes one of which has an odd atomic number. This, however, constitutes only  $\frac{1}{8}$  part of the mixture, so that the satellites arising from it have only a total intensity of 12% of the main line, which is due to the even isotopes. In krypton, as in nearly all other elements, there is no observed splitting of the lines of the even isotopes due to nuclear spin. Since the number of components into which the lines due to the odd isotope split is at least five (with the exception of four lines in the visible for which  $I = 1 \lesssim I = 0$ , which have only three components), it can be seen that the intensity of each component is very low. This is very advantageous for the use of krypton lines as wave-length standards, but it makes the analysis of the fine structure extremely difficult. The only lines which have ever been completely resolved are the four which have only three components. All the structures appear to be symmetrical.

The first work on the hyperfine structure of krypton was by Gehrcke and JANICKI\* who examined sixteen of the strongest lines with Lummer-Gehrcke plates. They found all the lines sharp and failed to detect structure in any of them.

More recently the hyperfine structure has been examined by Humphreys† and by Kopfermann and Wieth-Knudsen,‡ and finally I have made a careful examination of all the stronger lines in the violet part of the spectrum and several lines in the less refrangible part of the spectrum, i.e., all the lines whose wave-lengths are given in Table III.

Both Humphreys and Kopfermann and Wieth-Knudsen were unable to detect any structure in the violet lines of krypton with the aid of Fabry-Perot étalons, but in the hope of resolving some of them I examined them with the aid of a reflecting

<sup>\* &#</sup>x27;Ann. Physik,' vol. 81, p. 314 (1926).

<sup>† &#</sup>x27;Bur. Stand. J. Res.,' vol. 7, p. 453 (1931).

<sup>‡ &#</sup>x27;Z, Physik,' vol. 85, p. 353 (1933).

échelon.\* However, in no case could any measurable structure be detected, although the resolving power of the échelon was over 600,000. When the photographs were over-exposed about 100 times there was a suggestion of hazy satellites on each side of the main line, but it is quite possible that they were faint ghosts. In any case none of the violet lines shows any structure which could possibly impair its value as a wave-length standard. The green, yellow, and red lines were examined with étalons of  $1\frac{1}{2}$  and  $2\frac{1}{2}$  cm separation and an objective of rings of 1 metre focus, but only for the lines 5580 and 5570 was any measurable structure observed. results of my investigation, which did not include the infra-red region, together with those of Humphreys and Kopfermann and Wieth-Knudsen are given in Table III. The agreement is satisfactory throughout.

TABLE III—THE OBSERVED HYPERFINE STRUCTURE OF KRYPTON LINES.

λ	Jackson	Kopfermann and Wieth-Knudsen	Humphreys
5570	$+0.11~{ m cm^{-1}}\ -0.10~{ m hazy}$	$+0.111 \text{ cm}^{-1} \\ -0.120$	$+0.1073~{\rm cm^{-1}}$
5580	$+0\cdot 11 \\ -0\cdot 12$	Not observed	Not observed
5707	Has structure but too faint for measurement	,,	,,
5870	Broadened	, ,,	,,
7685	Not observed	$+0 \cdot 1135 \\ -0 \cdot 1225$	$+0.1135 \\ -0.1225$
8059	"	$+0.103 \\ -0.088$	$+0.1004 \\ -0.0879$
8263	"	$+0.135 +0.082 \ -0.112 -0.062$	Not observed
8281	"	$+0.130 \\ -0.145  -0.62$	$+0\cdot 142 \\ -0\cdot 145$
8509	"	$+0.127 +0.077 \\ -0.115$	$\begin{array}{cc} +0\cdot 1252 & +0\cdot 0871 \\ -0\cdot 1132 & \end{array}$

KOPFERMANN and WIETH-KNUDSEN have come to the conclusion that the nuclear spin of krypton cannot be less than 7/2, but it may very well be considerably higher than that.

None of the observers has been able to detect any isotope shift in the main lines due to the even isotopes, and even when the infra-red lines are self-reversed they remain perfectly symmetrical. This indicates that any isotope shift must be extremely small—far below the Doppler width of the lines.

<sup>\*</sup> Kindly placed at my disposal by my brother, Mr. D. A. JACKSON.

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It should be added for completeness, that Miss Romanova and Miss Ferchmin\* state that they have observed eight satellites of the line 5562, and six of 5649, but it is difficult to attach any importance to this statement, as most of the satellites they claim to have observed are well inside the Doppler width of the main line, even when a tube cooled with liquid air is used. It is also significant that they failed to observe the satellites of 5570 at +0.110 cm<sup>-1</sup> and -0.120 cm<sup>-1</sup>, which have been observed by Humphreys, Kopfermann and Wieth-Knudsen, and JACKSON.

The theory of the visibility of fringes according to the gas-kinetic laws and the Doppler-Fizeau principle has been given by Buisson and Fabry,† who show that the width of a line  $\Delta = 0.82 \times 10^{-6}$ 

$$\sqrt{\frac{\mathrm{T}}{m}}$$

where T is the absolute temperature of the gas,

m is the atomic (or molecular) weight of the radiating particle, and

 $\Delta = \frac{\lambda}{N}$  where N is the greatest number of waves which will give observable fringes.

The observed and calculated values of  $\Delta$  for the krypton lines observed in this investigation are given in the third column of Table IV. The values are probably accurate within one or two mA, and refer to low pressure tubes. When the pressure of krypton in the tube is 10 mm Hg the breadth of the 1s-3p lines is slightly (about 20%) broadened, but that of the 1s-2p lines is not appreciably It can be seen from the table that, with a few exceptions, the observed breadth of the lines does not exceed the calculated by more than the error of the The observed values were determined by estimating N from the appearance of the fringes when observed with étalons of 6,  $7\frac{1}{2}$ , and 10 cm thickness and correcting for their instrumental width.

#### The Wave-Lengths of the Krypton Lines

These have been derived from the measurement of about 120 plates by direct comparison with the primary standard. They refer to the wave-lengths emitted by tubes containing krypton at a pressure of 4 mm Hg, viewed either transversely or There was found to be no measurable difference between the wavelengths observed end-on or transversely. The results are presented in Table IV, the first column, headed \(\lambda\) (Jackson) gives the weighted mean of the wave-lengths

<sup>\* &#</sup>x27;C. R. U.S.S.R. Nouvelle Serie,' No. 2 (1933).

<sup>† &#</sup>x27;J. Phys. Rad.,' vol. 2, p. 442 (1912).

TABLE IV—THE OBSERVED WAVE-LENGTHS AND WIDTHS OF KRYPTON LINES GIVEN BY A TUBE FILLED AT A PRESSURE OF 4 MM HG, AND THE PRESSURE CORREC-TIONS FOR A TUBE AT VERY LOW PRESSURE.

	Breadth		Pressure	
λ (Jackson)	Obs Calc	$\operatorname{Terms}$	Correction	ν
3424·9720 (4) D	6 mA	$1s_5$ — $5p_6$	distribution.	29188 · 979
3431·7511 (5) C	6 mA	$1s_5$ — $5p_9$		29131 · 322
3495 · 9897 (2) C	$6~\mathrm{mA}$	$1s_5 - 3p_2$	$-0.00005 \mathrm{A}$	28596 · 051
3522·675 (4) E	$6~\mathrm{mA}$	$1s_4 - 5p_5$		$28379 \cdot 430$
3615·4749 (7) B	6  mA	$1s_4 - 3p_2$	-0.00005  A	$27651 \cdot 028$
3628·1571 (3) C	6 mA	1s <sub>4</sub> 5Y		27554 · 376
3665·3263 (19) B	6 mA	$1s_5 - 4p_6$	-0.0009  A	$27274 \cdot 959$
3668·7374 (3) C	6 mA	$1s_5 - 4p_7$	$-0.0009 \mathrm{A}$	$27249 \cdot 602$
3698·047 (1) E	6 mA	$1s_{5} - 4p_{10}$	$-0.0009 \mathrm{A}$	27033 · 493
3773·4247 (19) B	6 mA	$1s_4$ — $4p_5$	$-0.0009 \mathrm{A}$	26493 · 627
0500 0044 (0) 0			0.0000	
3796·8844 (8) C	7 mA	$1s_4$ — $4p_6$	-0.0009  A	26329 · 935
3800 · 5440 (13) B	7 mA	$1s_4 - 4p_7$	-0.0009  A	26304 · 582
3812·2159 (8) C	7 mA	$1s_4 - 4p_8$	$-0.0009 \mathrm{A}$	26224 · 047
3837 · 8162 (12) B	7 mA	$1s_5$ — $4Y$	-0.0010  A	26049 · 130
4273 · 9700 (97) A	8 mA	$1s_5$ — $3p_6$	-0.00006  A	23390 · 889
4282·9683 (82) B	8 mA	1s <sub>5</sub> 3p <sub>7</sub>	-0.00006  A	$23341 \cdot 748$
4286·4873 (25) B	$8 \mathrm{mA}$	$1s_3 - 3p_3$	$-0.00006 \mathrm{A}$	$23322 \cdot 586$
4300·4877 (16) B	8 mA	$1s_3 - 3p_4$	$-0.00006 \mathrm{A}$	$23246 \cdot 660$
4318·5525 (24) A	8  mA	$1s_5$ — $3p_8$	$-0.00006 \mathrm{A}$	$23149 \cdot 419$
4319·5796 (54) A	8 mA	$1s_5$ — $3p_9$	-0.00006  A	23143 · 916
4351·3609 (28) A	8 mA	$1s_2 - 3p_1$	-0.00007 A	22974 · 880
4362 · 6423 (103) A	8 mA	$1s_{5}$ — $3p_{10}$	-0.00007 A	$22915 \cdot 470$
4376·1220 (103) A	8 mA	$1s_4 - 3p_5$	$-0.00007 \mathrm{A}$	$22844 \cdot 886$
4399·9669 (76) A	$8  \mathrm{mA}$	$1s_2 - 3p_2$	-0.00007  A	$22721 \cdot 083$
4410·3687 (8) B	8 mA	$1s_2 - 3p_3$	-0·00007 A	$22667\cdot 497$
4418·764 (5) E	8 mA	1s <sub>2</sub> —5Y	Madelinia	22624 · 429
4425·1906 (16) C	8 mA	$1s_2 - 3p_4$	-0.00007  A	$22591 \cdot 575$
4453·9179 (100) A	8 mA	$1s_4 - 3p_6$	-0.00007  A	$22445 \cdot 863$
4463·6902 (107) A	8 mA	$1s_4 - 3p_7$	-0.00007  A	$22396 \cdot 724$
4502·3547 (103) A	8  mA	$1s_4$ — $3p_8$	-0.00007  A	$22204 \cdot 393$
4550·2985 (6) D	8 mA	$1s_4 - 3p_{10}$	-0.00007  A	21970 · 443
5562·2257 (55) A	9 mA	$1s_4$ $3p_{10}$ $1s_5$ $2p_2$	±0.0000 A	17973 · 436
5570 · 2895 (62) A	9 mA	$1s_{5}$ $2p_{2}$ $1s_{5}$ $2p_{3}$	$\pm 0.0000 \mathrm{A}$	17947 • 417
5580 · 388 (10) E*	9 mA	$1s_{2}$ — $3p_{5}$	$\pm 0.0000 \mathrm{A}$	17914 · 939
5649 · 5628 (17) B	9 mA	$1s_{2}$ — $3p_{5}$ $1s_{3}$ — $3p_{10}$	$-0.0001 \mathrm{A}$	17695 · 586
0010 0020 (II) D	O 1117 k	13 3 -Op 10	0 000111	17000.000

<sup>\*</sup> The measurement of this line was complicated by the proximity of the strong line at 5570.

Table IV—continued

	Brea	dth		Pressure	
λ (Jackson)	Obs	Calc	Terms	Correction	
5672·4519 (7) B	9 n	n ${f A}$	1s <sub>5</sub> 2p <sub>4</sub>	$\pm 0.0000  \mathrm{A}$	17624 · 182
5707·512 (1) E	10  mA	9  mA	$1s_2$ — $3p_6$	$-0.0001 \mathrm{A}$	$17515 \cdot 921$
5832·859 (3) E	10  mA	9  mA	$2p_{6}$ — $6d_{4}$	e	$17139 \cdot 512$
5870 · 9158 (56) A	13  mA	10  mA	$1s_4$ — $2p_2$	$\pm 0.0000  \text{A}$	$17028 \cdot 409$
5879·9000 (15) D	11 mA	10 mA	$1s_4$ — $2p_3$	$\pm 0.0000 \mathrm{A}$	1 <b>70</b> 02 · 391
5993·8503 (17) C	11 mA	10 mA	$1s_4$ — $2p_4$	$\pm 0.0000 \mathrm{A}$	16679 · 158
6012·1570 (11) D	11  mA	10 mA	$2p_{6}$ — $5s_{5}$	-0.0001  A	$16628 \cdot 279$
6056·1280 (13) D	11 mA	10 mA	$2p_{10}$ — $5d_5$	-0.0001  A	$16507 \cdot 640$
6082· (7)*	13  mA	10 mA	$2p_{10}$ — $5d_{6}$	-0.0001  A	
6236·354 (1) E	12 m <b>A</b>	10 mA	$2p_{9}$ — $4s_{5}$		$16030\cdot 585$
6421·0300 (13) D	12  mA	10 m <b>A</b>	$2p_{8}$ — $5d_{4}$	-0.0001 A	15569 · 531
6456·2910 (21) C	12 mA	10 mA	$2p_{9}$ — $5d_{4}$	-0.0001  A	$15484 \cdot 498$

This line shows variation in wave-length according to interferometer separation.

determined with the various étalons, and the total number of plates from which the wave-lengths are derived. The letters following the number of measurements of the lines indicate the estimated accuracy of the measurements and have the following significance:—

A	error	not greater	than	$0\!\cdot\!0001A$
В	,,	,,		$0\!\cdot\!0002A$
$\mathbf{C}$	<b>,</b> ,	,,		$0\!\cdot\!0003A$
D	,,	,,	4	0.0005A
$\mathbf{E}$	•	••		0.001A

The accuracy A is attributed only to lines measured on 30 or more plates, and B to lines measured on 10 or more plates, even if the agreement between the wavelength determinations implies the justification of the letters A or B. The second column gives the breadth of the line, observed and calculated. The third column gives the terms from which the line is derived, from the data of Meggers, De Bruin and Humphreys,\* while the fourth column gives the correction which should be applied to the wave-lengths, if a tube of low pressure is used instead of one filled at It can be seen that it is only for lines involving 4p terms that this is appreciable. These are all ultra-violet lines that are of somewhat low intensity, which are more suitable for use in conjunction with the other lines as standards for determining phase correction than as actual wave-length standards. The pressure correction even for these lines between a new tube of 4 mm Hg pressure and an old

<sup>\* &#</sup>x27;Bur. Stand. J. Res.,' vol. 7, p. 643 (1931).

one of unknown pressure does not exceed 0.001A—an amount quite insignificant when the wave-lengths are used only for determining phase correction.

It was found that the systematic differences between the measurements with étalons of various thicknesses, over the range  $\frac{1}{2}$  cm to 10 cm, are in no case greater than the experimental error. For étalons down to 3 cm thickness this is about one part in 50,000,000, while for shorter étalons it is correspondingly greater. gives very strong confirmation of the belief that the primary standard has a wavelength which is accurately reproducible and independent of the resolving power of the interferometer used. This naturally applies also to the krypton scale. extremely improbable that the cadmium line should exactly follow a mean change in wave-length of a large number of krypton lines covering the whole visible spectrum, and involving 20 different terms.

The lines which have been observed on one plate only are given to 7 figures only; three of the adopted wave-lengths have been given to 7 figures only, because the disagreement between the results from the various étalons does not justify the retention of the eighth figure, although it appears improbable that the error exceeds The inaccuracy in the case of these lines, which are all faint ones, is due to the fact that they were mostly measured on under-exposed plates, rather than to any faults in the lines themselves.

The values of the systematic differences between the six different sets of observations made with six étalon thicknesses are given in Table V.

Table V

	Systematic difference	,	Systematic difference
Étalon	Mean—Observed	Étalon	Mean—Observed
$\frac{1}{2}$ cm	$<~0\cdot 0005~\mathrm{A}$	5 cm	$< 0.0001 \mathrm{A}$
1 cm	$< 0.0002 \mathrm{A}$	$6~\mathrm{cm}$	< 0.0001  A
$2~\mathrm{cm}$	$< 0.0001  \mathrm{A}$	$7\frac{1}{2}$ cm -	$-0.0000_{5}A$
$3~\mathrm{cm}$	< 0.0001  A	10 cm	+ 0.0001  A

These figures indicate that the scale error of the krypton lines is very unlikely to exceed 0.0001A.

It has not been considered advisable to give the vacuum wave-numbers beyond the third decimal place because there is too much doubt as to the correct value of the vacuum correction. The vacuum wave-numbers were found by reducing the wave-lengths to vacuum, using the values given in Kayser's Tabelle der Schwingenzahlen (which were derived from Meggers's and Peters's data on the refractive index of air). The reciprocals of the vacuum wave-lengths were divided out on a calculating machine, and are given in the last column of Table IV.

Term values have been calculated for 30 terms, assuming that  $1s_5 = 32943 \cdot 165$ (after Meggers, de Bruin and Humphreys). These are given in Table VI.

From these it is possible to calculate wave-lengths for lines which have not been observed interferometrically. The fourth decimal place has not been retained as

it would have little meaning on account of the uncertainty of the vacuum correction. These wave-lengths are given in Table VII.

### TABLE VI—TERM VALUES OF THE SPECTRUM OF KRYPTON.

$1s_2 = 27068 \cdot 194$	$3p_8 = 9793.946$
$1s_3 = 27723 \cdot 281$	$3p_9 = 9799 \cdot 249$
$1s_4 = 31998 \cdot 138$	$3p_{10} = 10027.695$
$1s_5 = 32943 \cdot 165$	
	$4p_5 = 5504 \cdot 512$
$2p_2 = 14969 \cdot 728$	$4p_6 = 5668 \cdot 204$
$2p_3 = 14995 \cdot 747$	$4p_7 = 5693.559$
$2p_4 = 15318.981$	$4p_8 = 5774 \cdot 091$
	$4p_{10} = 5709 \cdot 672$
$3p_1 = 4093 \cdot 314$	
$3p_2 = 4347 \cdot 112$	$5p_5 = 3618.708$
$3p_3 = 4400.697$	$5p_6 = 3754 \cdot 186$
$3p_4 = 4476 \cdot 619$	$5p_9 = 3811.743$
$3p_5 = 9153 \cdot 253$	$4Y = 6894 \cdot 025$
$3p_6 = 9552 \cdot 276$	5Y = 4443.764
$3b_{-} = 9601.415$	

### TABLE VII—CALCULATED WAVE-LENGTHS IN THE SPECTRUM OF KRYPTON.

Jackson		M, deB, H	
calculated		Observed	Terms
3502 · 553	$28542\cdot 470$	$3502\cdot 56$	1s <sub>5</sub> 3p <sub>3</sub>
$3507 \cdot 846$	$28499 \cdot 401$	$3507 \cdot 84$	$1s_5$ — $5Y$
3511.895	$28466 \cdot 546$	$3511 \cdot 91$	$1s_5 - 3p_4$
$3539 \cdot 573$	$28243\cdot 952$	$3539\cdot 55$	$1s_4 - 5p_6$
$3622\cdot 495$	27597 · 441	$3622\cdot 53$	$1s_4 - 3p_3$
$3632 \cdot 489$	27521 · 519	$3632\cdot 53$	$1s_4 - 3p_4$
$3982 \cdot 289$	$25104 \cdot 113$	$3982 \cdot 18$	$1s_4$ —4Y
$4263 \cdot 290$	$23449 \cdot 486$	$4263 \cdot 29$	$1s_2 - 5p_5$
$4538 \cdot 054$	$22029\cdot 722$	4538.06	$1s_3 - 4p_7$
$4636 \cdot 133$	21563 · 682	4636 · 14	$1s_2$ — $4p_5$
4671 · 596	21399 • 990	4671 · 61	$1s_2 - 4p_6$
$4677 \cdot 094$	$21374 \cdot 635$	$4677\cdot 16$	$1s_2 - 4p_7$
$4694 \cdot 826$	$21294 \cdot 103$	$4694 \cdot 84$	$1s_2 - 4p_8$
$4955 \cdot 455$	$20174 \cdot 169$	$4955 \cdot 27$	$1s_2$ —4Y
5516 · 667	18121 · 866	5516 - 66	$1s_3 - 3p_7$
5787 · 296	$17274 \cdot 448$	5787 · 29	$1s_2 - 3p_8$
$5866 \cdot 750$	$17040 \cdot 499$	$5866 \cdot 74$	$1s_2 - 3p_{10}$
$7854 \cdot 825$	$12727 \cdot 533$	$7854 \cdot 823$	$1s_3$ — $2p_3$
$8059 \cdot 505$	$12404 \cdot 302$	$8059 \cdot 5053$	$1s_3 - 2p_4$
8263 · 239	$12098 \cdot 466$	$8263 \cdot 2412$	$1s_2-2p_2$
8508 • 873	11749 · 213	8508 · 8736	1s <sub>2</sub> -2p <sub>4</sub>

DETERMINATION OF THE THICKNESS OF AN ÉTALON BY MEANS OF THE KRYPTON LINES

If the ring system of one spectral line only be measured, the thickness of the étalon, t, is given by the equation

 $2t = (N + e)\lambda$ 

where N is the whole number of the order of interference and e is the fraction given by the measurement of the ring system.

We can evaluate 2t unambiguously only if the approximate value, which is usually measured by means of a screw micrometer, is known to within about  $\frac{1}{4}$  wave-length This corresponds to less than  $0.1 \mu$  and it is usually not possible to measure, by direct methods, the rough thickness of an étalon with this degree of accuracy. If, however, two, or preferably more, ring systems of lines of accurately known wave-length be measured it is possible to determine the whole number of waves contained by the étalon by a method which is actually a modification of the method of coincidences.

In the practical application of this method it is therefore necessary to have a spectrum which not only contains a fair number of lines whose wave-lengths are accurately known, but also whose distribution is suitable. The spectrum of krypton fulfils both these conditions so well that the evaluation of the thickness of an étalon (even when the approximate thickness is in error by 0.1 mm) is a very simple matter. For this reason it seems worth while to give a short account of the principle of the method and two examples of its application.

If the fractional parts of the orders of interference,  $e_1$  and  $e_2$ , have been determined for two lines of wave-length  $\lambda_1$  and  $\lambda_2$ , we have

 $(N_{1t} + n_1 - e_1) = 2t$ 

and

$$(\mathrm{N_2} + \left(n_1 \, rac{\lambda_1}{\lambda_2}
ight) + \mathit{e}_2) = 2\mathit{t}$$

where  $N_{1i} + n_1 = N_1$ , and  $N_1$  and  $N_2$  are the correct values for the whole numbers of the orders of interference, for  $\lambda_1$  and  $\lambda_2$  respectively.

If, therefore, the fractional part of the order of interference for  $\lambda_2$  be calculated from the measured thickness of the étalon  $(N_{1t}+e_1)$  it will only be in agreement with the observed value if  $n_1 = 0$ , i.e., if the approximate value  $N_{1i}$ , which is obtained by dividing the measured thickness of the étalon by  $\frac{1}{2}\lambda_1$ , is correct. If  $N_1$ , is incorrect the calculated value for the fractional order of interference of the ring system of  $\lambda_2$ 

will differ from the observed value by the quantity  $n_1 - \left(n_1 \frac{\lambda_1}{\lambda_2}\right)$  or  $n_1 \left(1 - \frac{\lambda_1}{\lambda_2}\right)$ .

We thus have,

$$e_{2 \text{ calculated}} = e_{2 \text{ observed}} - n_1 \left(1 - \frac{\lambda_1}{\lambda_2}\right)$$

which gives the value of  $n_1$ .

The factor  $\left(1-\frac{\lambda_1}{\lambda_2}\right)$  is a constant for any given pair of lines irrespective of the

value of N. For this reason much labour may be saved by compiling a short table giving the values for  $n_1$  for various values of  $(e_{2\text{calc}} - e_{2\text{obs}})$  for the pairs of lines most suitable for the evaluation of the thickness of an étalon. These values are given in Table VIII for the most suitable violet lines and in Table IX for the visible lines of krypton.

It can be seen from Table VII that the pair 4318 and 4319 enable one to determine the thickness of an étalon if  $n_1$  is not greater than about 1500 (which corresponds to an error in 2t of 0.3 mm) with an accuracy of  $\pm$  50. The pair 4273 and 4318 may then be used for values of  $n_1$  up to about 100 with an accuracy of about  $\pm 5$ , while the pair 4273 and 4502 may then be used to find  $n_1$  with an accuracy of at least 0.2. Since  $n_1$  is necessarily an integer, this gives an unambiguous value for it.

Two examples of the determination of the exact thickness of étalons will make the method quite clear.

For the first example let us consider an étalon of 3 cm thickness, whose approximate dimensions have been measured with a vernier caliper, giving a possible error of about  $\pm 0.1$  mm for 2t. This corresponds to two or three hundred wavelengths, and it is therefore necessary to use the close pair 4318 and 4319 to get a better preliminary value for 2t.

The measured thickness was 2t = 60.0 mm, which means that for the line 4318,  $N_{1i} = 139000 \pm 200$ . The observed value of e for this line was 0.450 and we shall therefore assume temporarily that  $2t_1 = 139000 \cdot 460 \times 4318 \cdot 552$ .

Thus we have,

$$\lambda ext{ standard } e_{ ext{obs}} ext{ } 2t_1/\lambda ext{ } e_{ ext{cal}} - e_{ ext{obs}} \ 4318 \cdot 5525 ext{ } 0 \cdot 460 ext{ } 139000 \cdot 460 ext{ } - \ 4319 \cdot 5796 ext{ } 0 \cdot 424 ext{ } 138971 \cdot 457 ext{ } + 0 \cdot 034 \ ext{ }$$

From Table VIII we see that  $n = +150 \pm 20$ 

Therefore we shall now take  $2t = 139150 \cdot 460 \times 4318 \cdot 5525$ = 60.0928 mm.

With this value for the thickness of the étalon we can now calculate the exact number of waves contained by it, by using the lines 4273, 4318, 4362, and 4502.

By dividing 60.0928 by 4273.9700, we find that the approximate number of waves of this radiation contained by the étalon is 140,601. We shall therefore start by assuming that,

 $2t_2 = 4273 \cdot 9700 \times 140601 \cdot 580 = 60 \cdot 0926935$ 

$\lambda$ standard	$e_{ m obs}$	$2t_{\scriptscriptstyle 2}/\lambda$	$e_{ m cal} - e_{ m obs}$	$2t_3/\lambda$	$\ell_{\rm cal} - \ell_{\rm obs}$	λ observed
$4273 \cdot 9700$	0.580	140601 · 580		140563 · 580		$4273 \cdot 9700$
$4282 \cdot 9683$	0.270	$140306 \cdot 186$	-0.084	140268 · 263	-0.007	$4282 \cdot 9681$
$4318 \cdot 5525$	0.460	$139150 \cdot 082$	-0.378	139112 · 474	+0.014	4318.5530
$4362 \cdot 6423$	0.580	$137743 \cdot 805$	-0.775	137706 · 574	-0.006	$4362 \cdot 6421$
$4502 \cdot 3547$	0.413	$133469 \cdot 478$	-1.935	133433 · 407	-0.006	$4502 \cdot 3545$
$5562 \cdot 2257$	0.950	$108037 \cdot 134$	-8.816	108007 • 937	-0.014	$5562 \cdot 2250$

Table VIII—Values of  $n_1$  for Pairs of Krypton Violet Lines.

$$\lambda_1 = 4318 \cdot 5525, \quad \lambda_2 = 4319 \cdot 5797, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0.000232$$

200 300 400 500 600 700 800  $0\cdot 002\ 0\cdot 023\ 0\cdot 046\ 0\cdot 070\ 0\cdot 093\ 0\cdot 116\ 0\cdot 139\ 0\cdot 162\ 0\cdot 185\ 0\cdot 208\ 0\cdot 232\ 0\cdot 348$ 

$$\lambda_1 = 4273.9700, \quad \lambda_2 = 4282.9683, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0.00210$$

20 30 40 50 60 70 80  $0\cdot 002\ 0\cdot 021\ 0\cdot 042\ 0\cdot 063\ 0\cdot 084\ 0\cdot 105\ 0\cdot 126\ 0\cdot 147\ 0\cdot 168\ 0\cdot 189\ 0\cdot 210\ 0\cdot 420$ 

$$\lambda_1 = 4273.9700, \quad \lambda_2 = 4318.5525, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0.01032$$

 $\frac{1}{0\cdot010}\,\frac{2}{0\cdot021}\,\frac{3}{0\cdot031}\,\frac{4}{0\cdot041}\,\frac{5}{0\cdot052}\,\frac{6}{0\cdot062}\,\frac{7}{0\cdot072}\,\frac{8}{0\cdot082}\,\frac{9}{0\cdot093}\,\frac{10}{0\cdot103}\,\frac{20}{0\cdot206}\,\frac{30}{0\cdot310}\,\frac{40}{0\cdot411}$ 

$$\lambda_1 = 4273.9700, \quad \lambda_2 = 4319.5797, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0.01056$$

 $0\cdot 010\ 0\cdot 021\ 0\cdot 031\ 0\cdot 042\ 0\cdot 053\ 0\cdot 063\ 0\cdot 074\ 0\cdot 084\ 0\cdot 095\ 0\cdot 106\ 0\cdot 211\ 0\cdot 316\ 0\cdot 422$ 

$$\lambda_1 = 4273.9700, \quad \lambda_2 = 4362.6423, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0.02032$$

 $\frac{1}{0\cdot020}\,\frac{2}{0\cdot041}\,\frac{3}{0\cdot061}\,\frac{4}{0\cdot081}\,\frac{5}{0\cdot102}\,\frac{6}{0\cdot122}\,\frac{7}{0\cdot142}\,\frac{8}{0\cdot162}\,\frac{9}{0\cdot183}\,\frac{10}{0\cdot203}\,\frac{20}{0\cdot406}\,\frac{30}{0\cdot610}\,\frac{40}{0\cdot813}$ 

$$\lambda_1 = 4273.9700, \quad \lambda_2 = 4502.3547, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0.050725$$

 $e_{2\text{calc}} - e_{2\text{obs}}$ 

$$\lambda_1 = 4273.9700, \quad \lambda_2 = 5562.2257, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0.23160$$

 $0\cdot 232\ 0\cdot 463\ 0\cdot 695\ 0\cdot 927\ 1\cdot 158\ 1\cdot 390\ 1\cdot 622\ 1\cdot 853\ 2\cdot 074\ 2\cdot 316\ 4\cdot 632\ 6\cdot 948\ 9\cdot 272$ 

$$\lambda_1 = 4453.9179$$
,  $\lambda_2 = 4463.6902$ ,  $1 - \frac{\lambda_1}{\lambda_2} = 0.002213$ 

20 30 50  $0\cdot 002 \quad 0\cdot 022 \quad 0\cdot 044 \quad 0\cdot 066 \quad 0\cdot 088 \quad 0\cdot 111 \quad 0\cdot 132 \quad 0\cdot 154 \quad 0\cdot 176 \quad 0\cdot 198$ 

$$\lambda_1 = 4453.9179$$
,  $\lambda_2 = 4502.3547$ ,  $1 - \frac{\lambda_1}{\lambda_2} = 0.01076$ 

 $\frac{1}{0\cdot011}\,\frac{2}{0\cdot022}\,\frac{3}{0\cdot032}\,\frac{4}{0\cdot043}\,\frac{5}{0\cdot054}\,\frac{6}{0\cdot065}\,\frac{7}{0\cdot075}\,\frac{8}{0\cdot086}\,\frac{9}{0\cdot098}\,\frac{10}{0\cdot108}\,\frac{20}{0\cdot215}\,\frac{30}{0\cdot323}\,\frac{40}{0\cdot430}$ 

$$\lambda_1 = 4453 \cdot 9179, \quad \lambda_2 = 5562 \cdot 2257, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0 \cdot 19256$$

 $0\cdot 193\ 0\cdot 385\ 0\cdot 578\ 0\cdot 770\ 0\cdot 963\ 1\cdot 155\ 1\cdot 348\ 1\cdot 540\ 1\cdot 733\ 1\cdot 926\ 3\cdot 851\ 5\cdot 777\ 7\cdot 702$ 

From Table VIII we see that

4273 and 4282 give 
$$n = -40 \pm 5$$
  
4273 ,, 4318 ,,  $n = -37 \cdot 8 \pm 1$   
4273 ,, 4362 ,,  $n = -38 \cdot 2 \pm 0 \cdot 5$   
4273 ,, 4502 ,,  $n = -38 \cdot 1 \pm 0 \cdot 2$   
4273 ,, 5562 ,,  $n = -38 \cdot 03 \pm 0 \cdot 05$ 

This shows that  $2t_3 = 4273.9700 \times 140563.580$  without any ambiguity. As a matter of interest, in showing the accuracy attainable from typical one plate measurements the wave-lengths resulting from this plate are given in the last column. The error in the case of 5562 is due to the fact that no phase correction or N.T.P. correction has been applied, while that of 4318 was caused by the partial overlapping of the fringes of the very strong line at 4319.

For the second example let us consider a 10 cm étalon for which it is known, from measurements with a screw micrometer, that  $2t = 200 \cdot 129 \text{ mm } \pm 0.01 \text{ mm}$  which gives  $N_t = 359800 \cdot 20$ . Since we know from the observations on the rings that e for  $\lambda 5562 = \cdot 100$ , we shall assume

$$2t_1 = 359800 \cdot 100 \times 5562 \cdot 2257 = 200 \cdot 128963$$

and we have then,

$\lambda$ standard	$\ell_{\rm obs}$	$2t_1/\lambda$	$e_{\mathrm{cal}} - e_{\mathrm{obs}}$	$2t_2/\lambda$	$e_{\rm cal} - e_{\rm obs}$	$\lambda$ observed
$5562 \cdot 2257$	$0 \cdot 10$	359800 · 10	**************************************	359790 · 10	· · · · · · · · · · · · · · · · · · ·	$5562 \cdot 2257$
$5570 \cdot 2895$	$0 \cdot 24$	$359279 \cdot 24$	-0.00	359269 · 25	+0.01	$5570 \cdot 2894$
$5649 \cdot 5630$	$0 \cdot 07$	$354237 \cdot 91$	-0.16	354228 · 06	-0.01	5649 · 5631
$5993 \cdot 8503$	0.18	$333890 \cdot 45$	-0.73	333881 · 17	+0.01	5993 · 8501
$6438 \cdot 4696$	0.43	$310833 \cdot 08$	$-1\cdot34$	$310824 \cdot 44$	+0.01	$6438 \cdot 4698$

From Table IX we see that

5562 and 5570 give 
$$n = 0 \pm 20$$
  
5562 ,, 5649 ,,  $n = -11 \pm 2$   
5562 ,, 5993 ,,  $n = -10 \pm 0.4$   
5562 ,, 6438 ,,  $n = -9.9 \pm 0.2$ 

This shows that  $2t_2 = 5562 \cdot 2257 \times 359790 \cdot 10 = 200 \cdot 123374$  mm. In this case the values of e were rounded off to the nearest hundredth, as the error (less than 0.0001 A) introduced by this procedure was not large enough to introduce any difficulty in the determination of the thickness of the étalon. The small errors in the observed values of the wave-lengths give an idea of the accuracy of interferometric comparisons of wave-length with long étalons for sharp lines. The results are not corrected to N.T.P. or for phase change. These corrections would range from 0.0002 A to *nil*.

Table IX—Values of  $n_1$  for Pairs of Krypton Visible Lines.

$$\lambda_1 = 5562 \cdot 2257, \quad \lambda_2 = 5570 \cdot 2895, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0 \cdot 001443$$
 
$$n_1 \qquad \qquad 1 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \quad 70 \quad 80 \quad 90 \quad 100 \quad 200$$
 
$$e_{2\text{calc}} - e_{2\text{obs}} \qquad 0 \cdot 001 \quad 0 \cdot 014 \quad 0 \cdot 029 \quad 0 \cdot 043 \quad 0 \cdot 058 \quad 0 \cdot 072 \quad 0 \cdot 087 \quad 0 \cdot 101 \quad 0 \cdot 115 \quad 0 \cdot 130 \quad 0 \cdot 144 \quad 0 \cdot 289$$

$$\lambda_1 = 5562 \cdot 2257, \quad \lambda_2 = 5649 \cdot 5628, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0 \cdot 01545$$

$$a_1 = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 20 \quad 30 \quad 40$$

$$a_{2\text{calc}} - e_{2\text{obs}} \quad 0 \cdot 015 \quad 0 \cdot 031 \quad 0 \cdot 046 \quad 0 \cdot 062 \quad 0 \cdot 077 \quad 0 \cdot 093 \quad 0 \cdot 108 \quad 0 \cdot 124 \quad 0 \cdot 139 \quad 0 \cdot 154 \quad 0 \cdot 309 \quad 0 \cdot 463 \quad 0 \cdot 618$$

$$\lambda_1 = 5562 \cdot 2257, \quad \lambda_2 = 5672 \cdot 4519, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0 \cdot 01943$$

$$n_1 \qquad 1 \qquad 2 \qquad 3 \qquad 4 \qquad 5 \qquad 6 \qquad 7 \qquad 8 \qquad 9 \qquad 10 \qquad 20 \qquad 30 \qquad 40$$

$$\frac{1}{2 \text{calc}} - e_{2 \text{obs}} \qquad 0 \cdot 019 \ 0 \cdot 039 \ 0 \cdot 058 \ 0 \cdot 078 \ 0 \cdot 097 \ 0 \cdot 117 \ 0 \cdot 136 \ 0 \cdot 155 \ 0 \cdot 175 \ 0 \cdot 194 \ 0 \cdot 389 \ 0 \cdot 583 \ 0 \cdot 777$$

$$\lambda_1 = 5562 \cdot 2257, \quad \lambda_2 = 5870 \cdot 9158, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0 \cdot 05258$$
 
$$n_1 \qquad \qquad 1 \qquad 2 \qquad 3 \qquad 4 \qquad 5 \qquad 6 \qquad 7 \qquad 8 \qquad 9 \qquad 10 \qquad 20 \qquad 30 \qquad 40$$
 
$$e_{2\text{calc}} - e_{2\text{obs}} \qquad 0 \cdot 053 \ 0 \cdot 105 \ 0 \cdot 158 \ 0 \cdot 210 \ 0 \cdot 263 \ 0 \cdot 315 \ 0 \cdot 368 \ 0 \cdot 421 \ 0 \cdot 473 \ 0 \cdot 526 \ 1 \cdot 052 \ 1 \cdot 577 \ 2 \cdot 103$$

$$\lambda_1 = 5562 \cdot 2257, \quad \lambda_2 = 5993 \cdot 8503, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0 \cdot 07201$$

$$n_1 \qquad 1 \qquad 2 \qquad 3 \qquad 4 \qquad 5 \qquad 6 \qquad 7 \qquad 8 \qquad 9 \qquad 10 \qquad 20 \qquad 30 \qquad 40$$

$$e_{2\text{calc}} - e_{2\text{obs}} \qquad 0 \cdot 072 \ 0 \cdot 144 \ 0 \cdot 216 \ 0 \cdot 288 \ 0 \cdot 360 \ 0 \cdot 432 \ 0 \cdot 504 \ 0 \cdot 576 \ 0 \cdot 648 \ 0 \cdot 720 \ 1 \cdot 440 \ 2 \cdot 160 \ 2 \cdot 880$$

$$\lambda_1 = 5562 \cdot 2257, \quad \lambda_2 = 6456 \cdot 2910, \quad 1 - \frac{\lambda_1}{\lambda_2} = 0 \cdot 13848$$

$$n_1 \qquad 1 \qquad 2 \qquad 3 \qquad 4 \qquad 5 \qquad 6 \qquad 7 \qquad 8 \qquad 9 \qquad 10 \qquad 20 \qquad 30 \qquad 40$$

$$\ell_{\text{scale}} - \ell_{\text{sobs}} \qquad 0 \cdot 138 \quad 0 \cdot 277 \quad 0 \cdot 415 \quad 0 \cdot 554 \quad 0 \cdot 692 \quad 0 \cdot 831 \quad 0 \cdot 969 \quad 1 \cdot 108 \quad 1 \cdot 246 \quad 1 \cdot 385 \quad 2 \cdot 770 \quad 4 \cdot 154 \quad 5 \cdot 539$$

In conclusion it is a great pleasure to thank Professor A. Fowler for his great interest and the encouragement he has given me throughout the course of this work.

#### SUMMARY

The wave-lengths of forty-seven of the brightest lines in the first spectrum of krypton in the range  $\lambda 3424-\lambda 6456$  have been compared directly with the primary standard. For this purpose two new types of étalon were designed and a new type of étalon carrier was used. Étalons of thickness  $\frac{1}{2}$ , 1, 2, 3, 5, 6,  $7\frac{1}{2}$ , and 10 cm were used for the measurements, which are based on about 120 direct comparisons with the primary

standard. It was found that, within experimental error, there was no systematic variation of the wave-lengths of the krypton lines, relative to the primary standard. This implies constancy of apparent wave-length for the krypton lines and for the red cadmium line. The majority of the wave-lengths are thought to be accurate to 0.0001 or 0.0002 A.

Term values have been calculated for 28 terms in the first spectrum of krypton. This has rendered possible the calculation of wave-lengths for 21 lines which were not observed interferometrically.

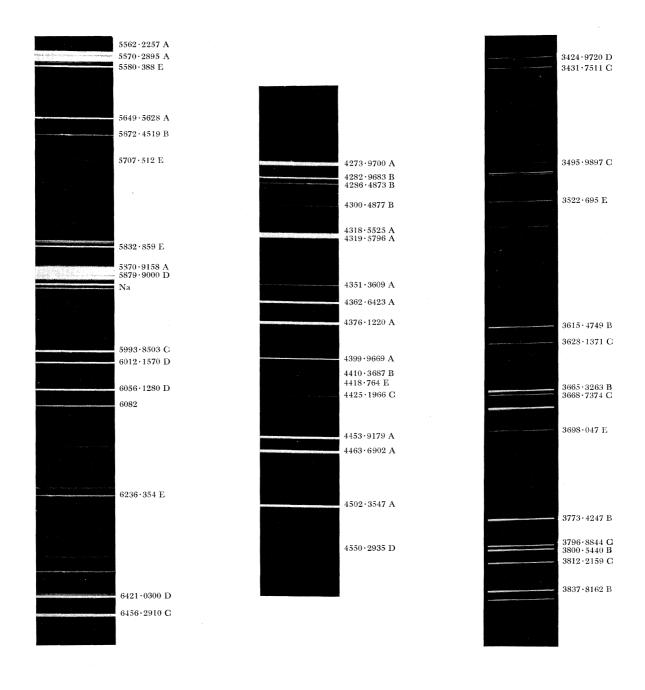
It has been found that, within experimental error, all the stronger lines given in the list have the same wave-length whether the tube be observed end-on or transversely, and that the presence of considerable quantities of impurity in the tubes has no measurable influence on the wave-lengths of the krypton lines.

The pressure shifts of 30 of the lines have been directly measured, using tubes filled at 0·1, 4, 10, and 20 mm Hg. From these results pressure shifts for many of the terms of the KrI spectrum have been calculated; which renders possible the calculation of the pressure shifts for nearly all the lines whose wave-lengths have been measured. This is of importance for deciding the pressure at which to fill the With the exception of a few lines in the ultra-violet no appreciable shift is introduced if the pressure does not exceed 4 mm Hg.

It is shown that the krypton spectrum is very convenient for the determination of the thickness of an étalon, even when the rough value of 2t is in error by as much as  $0.2 \,\mathrm{mm}$ . A simple method for the application is given with examples and tables of certain constants, which are useful for reducing the time required for making calculations.

Jackson

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	5562·2257 A 5570·2895 A 5580·388 E		3424·9720 D
	2300.309 F		3431·7511 C
	5649 · 5628 A		
	5672 · 4519 B		
	5707·512 E	4273 · 9700 A	3495 · 9897 C
		4282 · 9683 B 4286 · 4873 B	
		4300 · 4877 B	3522 · 695 E
	5832·859 E	4318·5525 A 4319·5796 A	
	5870 · 9158 A		
	Downloaded from rsta.royalsocietypublishing.org	4351·3609 A	
		4362 · 6423 A	
		4376·1220 A	3615 · 4749 B
HEMATICAL, SICAL VGINEERING NCES	5993·8503 C		3628·1371 C
	6012·1570 D	4399 · 9669 A	
YYALA	6056·1280 D	4410 · 3687 B 4418 · 764 E	3665·3263 B
	6082	4425·1966 C	3668·7374 C
OPHICAL T		4453·9179 A	3698·047 E
PHILOSO TRANSA		4463 · 6902 A	
	6236·354 E		
MATHEMATICAL PHYSICAL & ENGINEERING SCIENCES	0230·334 E	4502·3547 A	0770 1017 D
			3773·4247 B
ETY		4550 · 2935 D	3796 · 8844 C 3800 · 5440 B
TRANSACTIONS SOCIETY OF OF			3812·2159 C
SACTIONS OF	6421·0300 D		3837·8162 B
PHILO	6456·2910 C		